

RECTANGULAR PULSE TRAIN- FOURIER SERIES

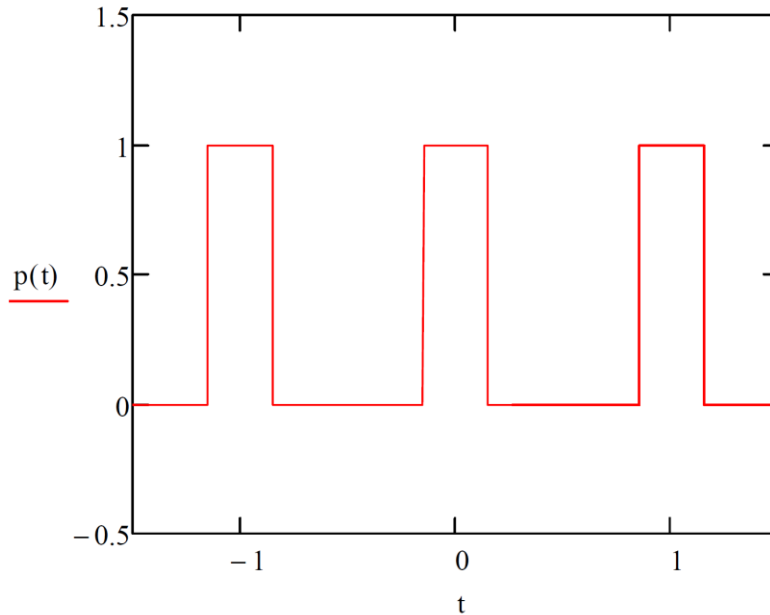
http://brewer.ece.gatech.edu/ece3043/Lecture_Notes/spectra.pdf

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$$x(t) = \begin{cases} A & |t| \leq \frac{\tau}{2} \\ 0 & \frac{\tau}{2} < t < \frac{T}{2} \\ x(t \pm nT) & n \text{ any integer} \end{cases}$$

so the duty cycle is $d = \tau/T$.



Pulse Train with Duty Cycle 0.15.

The Fourier expansion coefficients are

$$c_n = Ad \frac{\sin(\pi nd)}{\pi nd}$$

The expansion coefficient c_n are complex constants which can be determined from $x(t)$ as

$$c_n = \frac{1}{T} \int_{\alpha}^{\alpha+T} x(t) \exp(-jn\omega_p t) dt$$

Thus, the FOURIER Series expansion is

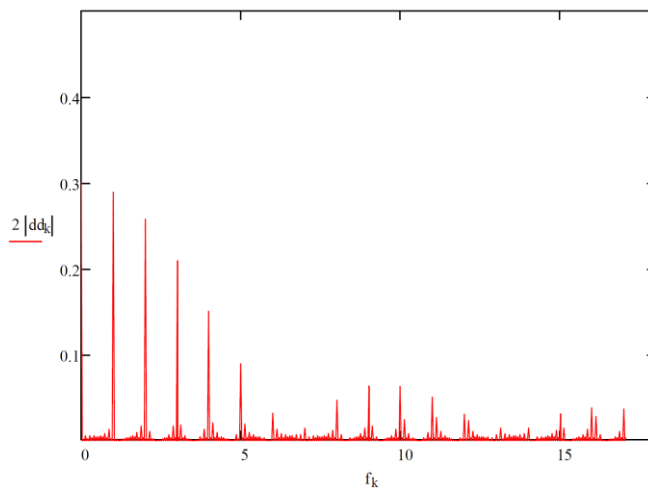
$$x(t) = \sum_{n=-\infty}^{\infty} c_n \exp(jn\omega_p t)$$

where $\omega_p = 2\pi f_p$ and $f_p = 1/T$ is the frequency in Hertz of the periodic function.

Since this is the complex exponential form of the series, the sum is from -infinity to infinity.

Note: The Duty Cycle changes the AMPLITUDE of the frequency components – NOT the frequency spectrum. In this example, the period $T = 1$ sec. The frequencies are thus

$F = [1 \ 2 \ 3 \ \dots \ n \ \dots] \text{ Hz}$ The amplitudes will be zero if $\pi n d = k\pi$ where the sine is zero in c_n . Never in this case if $d = 0.15$. If $d=0.5$, c_n is zero at $2k\pi$.



Spectra of Pulse Train

which is, of course, a line spectra but the envelope of the spectra has a $\sin(x)/x$

Here the magnitudes are $|2c_n|$ for the POSITIVE SPECTRUM. The “wiggles” at the base of the spectrum have to do with the plotting function for the spectrum- numerical noise.

FULL WAVE RECTIFIED SIGNAL – EXPONENTIAL FOURIER SERIES AS IN DSPF BOOK.

[Adam Panagos](#)

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<http://adampanagos.org> In this video we compute the exponential Fourier (EFS) series of a fully rectified sine wave signal $\sin(t)$. This computation involves computing the EFS coefficients D_n by projecting the signal onto the the n th exponential basis signal.

<https://www.youtube.com/watch?v=FIKPIRsADL0&list=RDCMUCvpWRQzhm8cE4XbzEHGth-Q&index=2>

Complex Fourier Series 38,068 views 15:56

Converts between Trig Series and Complex Series.

<https://www.youtube.com/watch?v=Ft5iyapkSqM>

Complex Fourier Series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nd + \sum_{n=1}^{\infty} b_n \sin nd = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2} (e^{ind} + e^{-ind}) + \sum_{n=1}^{\infty} \frac{b_n}{2i} (e^{ind} - e^{-ind})$$
$$= \frac{a_0}{c_0} + \sum_{n=1}^{\infty} \underbrace{\left(\frac{a_n - ib_n}{2} \right)}_{c_n} e^{ind} + \sum_{n=-\infty}^{-1} \underbrace{\left(\frac{a_n + ib_n}{2} \right)}_{c_n} e^{ind}$$
$$= \sum_{n=-\infty}^{\infty} c_n e^{ind} \quad c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ind} dx$$
$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{ind}$$

Play (k) http://em.ma

11:24 / 15:56 CC HD

Test your Ears – From 100 Hz to 12kHz. Can you hear the difference between a square wave and a sine wave? 5:35

<https://www.youtube.com/watch?v=uluJWS2uvY>

<http://www.audiomasterclass.com> - A comparison between square waves and sine waves of various frequencies, displayed on an oscilloscope, with commentary.

Taylor series | Essence of calculus, chapter 11 22:20 (e^x at Time 13:30)

<https://www.youtube.com/watch?v=3d6DsjlBzJ4>

3BLUE1BROWN SERIES S2 • E11

More Exotic:

Understanding the Uncertainty Principle with Quantum Fourier Series | Space Time 14:49

The humble sound wave explains Heisenberg's Uncertainty Principle. PBS Space Time

<https://www.youtube.com/watch?v=izqaWyZsEtY>